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Numerical modelling of the hybrid rocket engine performance

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Abstract The purpose of this paper is to present method of the numerical modeling of the hybrid rocket engine's work as a tool for designing engines of this type. The model is intended as an accurate and simple to use development tool for use in preliminary design stages of hybrid rocket engines, improving the effectiveness and quality of this process. General assumptions underlying the use of a model are presented, together with an analysis of past work in this field. Results of an extensive experimental campaign are presented and compared with the results of numerical modelling in order to calibrate the proposed model and evaluate its accuracy. Parameter variation and optimization were conducted, proving functionality of the methodology. Presented numerical calculations show that the adopted approach to reduce the analysis time and complexity was correct. This method of numerical calculation of hybrid engine working parameters combines aspects of accuracy and simplicity at the early design stage to avoid time-consuming and costly changes in subsequent detailed stages of design.

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1. Introduction

The enormous increase in interest in satellite devices and their operating techniques is one of the main factors forcing the search for a suitable, tailor-made rocket propulsion. The scope of requirements includes not only the use of rocket engines for the propulsion of classic space rockets, but also...
more and more often for the propulsion of spacecraft, lunar vehicles and landers, as well as the correction and transfer of the orbit of artificial satellites. A new trend is the so-called space tourism, which in the future may play an increasingly important role and have a significant impact on the development of rocket propulsion.

In recent years, significant development of the private sector of the space industry has also been noticed - not only in the form of supporting state programs (construction of satellites, research equipment, etc.), but also as an independent supplier of launch rockets [1]. This trend is associated with placing greater emphasis on the economics and safety of the solutions used, in contrast to the so-far dominant priority - high efficiency. The above criteria are met by hybrid rocket engines, combining the advantages of liquid fuel engines [2,3]:

- the ability to control the engine thrust and its emergency shutdown,
- relatively high specific impulse, and engines for solid propellant,
- a relatively simple construction,
- low production cost.

The above-mentioned features have caused a large increase in interest in hybrid engines in recent years - they were used to drive, among others, Spaceship One and ILR 33 “Amber” rockets.

Scheme of the hybrid rocket engine in the simplest form is shown in Figure 1. The engine consists of a combustion chamber containing solid fuel grain, a liquid oxidizer installation (usually N₂O or HTP (high-test peroxide)) containing an oxidizer tank, pipes supplying the medium to the combustion chamber and the injector.

Despite a relatively simple structure, the phenomena occurring in hybrid rocket engines are complex and their description requires knowledge in many scientific disciplines such as chemistry, thermodynamics, aerodynamics or mechanics. This means that the creation of an accurate simulation of phenomena occurring in a hybrid rocket engine is a multi-step task requiring the use of complex multidimensional mathematical models of individual subsystems. The literature describes many 0D solutions [4–6], based on simple thermodynamic relationships supported by empirical data, but their accuracy is often not satisfactory [6,7]. 2D and 3D numerical simulations of flow and combustion processes in a hybrid engine [8–10] usually require high computational power, which can significantly extend the initial design period. Moreover, it is not always possible to predict a priori all the phenomena occurring in the engine, even after performing a time-consuming calibration with experimental data.

The above conclusions lead to the creation of software that would be computationally flexible and at the same time sufficiently accurate to allow for the initial design and prediction of the behaviour of the hybrid rocket engine.

2. Design premises

This publication presents the process of mathematical modelling of the hybrid rocket engine operating parameters (in the context of creating a preliminary design of a rocket engine). During the development of the discussed model, a

![Figure 1](http://repo.pw.edu.pl/) Scheme of the hybrid rocket engine.
thorough analysis has been performed in an effort to apply already existing solutions, while eliminating their errors and simplifications. The following steps regarding the content of the model were taken:

- modelling of the oxidizer tank blowdown,
- employing multiphase oxidizer injector model,
- employing solid fuel regression model based on the enthalpy balance, supported by empirical data,
- simulating complex fuel grain geometries.

A block diagram of the developed computational algorithm, divided into individual engine sections is shown in Figure 2.

3. Mathematical model of a hybrid rocket engine

3.1. Oxidizer tank blowdown model

The mathematical model developed for simulating the oxidizer tank blowdown (Figure 3) and self-pressurization is based on the following assumptions:

- the gaseous nitrous oxide present in the tank is an ideal gas,
- the liquid and gas phases are in a state of continuous equilibrium (liquid nitrous oxide evaporation is instantaneous),
- the pressure in the entire tank is equal to the pressure of the gas phase,
- the external walls of the oxidizer tank are adiabatic (no heat exchange with environment),
- gas phase, liquid phase and tank are at the same temperature (immediate heat exchange inside the tank).

The above assumptions allowed for the creation of a simple algorithm that could be a starting point for more complex models (for example, treating nitrous oxide as a real gas [11]).

The simulated tank was treated as a single control volume, which includes the liquid phase, the gas phase, and the tank itself. As a starting point for the development of the model, the following equations were chosen:

\[
\frac{dm_l}{dt} = -\dot{m}_{inj} \tag{1}
\]

\[
\frac{d}{dt}(U_g + U_l + U_t) = -H_{inj} \tag{2}
\]

\[
P_g V_g = m_g RT_t \tag{3}
\]

The volume of the tank is constant during its emptying, and the tank is filled only with nitrous oxide (both in the liquid and gaseous phase) during the whole engine operation time, consequently:

\[
V = V_g + V_l \tag{4}
\]

Eqs. (1)–(4) have been used to create a system of three differential equations, in which the only unknowns are the values of \(dm_l/dt\), \(dm_g/dt\), and \(dT_t/dt\):
The values of nitrous oxide vapour pressure and other thermodynamic and material constants are determined by empirical relations [12].

3.2. Oxidizer injector model

Determining the expected oxidizer mass flow rate is one of the most important tasks in the process of designing and simulating a hybrid rocket engine and necessitates the creation of an appropriately accurate mathematical model. Most of the nitrous oxide orifice flow models presented in the literature can be divided into two main categories [13,14]:

- single-phase flow models (eg. SPI),
- two-phase flow models (eg. HEM).

Because of its simplicity, in many publications on simulating hybrid rocket engines, a single-phase model is used:

\[
\dot{m}_{SPI} = C_d A_{inj} \sqrt{2 \rho_l (P_l - P_{ch})}
\]  

(8)

It is essentially accurate for small pressure drops across the injector - in such conditions choked flow and cavitation do not occur. In the model discussed, it was decided to use a more universal Dyer model [15], which is a combination of a one- and two-phase flow models:

\[
\dot{m}_{Dyer} = \frac{\dot{m}_{SPI} + \dot{m}_{HEM}}{1 + \kappa}
\]

(9)

where:

\[
\dot{m}_{HEM} = C_d A_{inj} \rho_2 \sqrt{2(h_l - h_{ch})}
\]

(10)

\[
\kappa = \sqrt{\frac{P_l - P_{ch}}{P_{sat} - P_{ch}}}
\]

(11)

In Figure 4 one can compare the results of nitrous oxide mass flow rate calculations through an injector with a given geometry and a set pressure at the orifice inlet (50 atm) depending on the outlet pressure, obtained using the discussed one- and two-phase models and the Dyer model.

3.3. Solid fuel grain regression model

The mechanism of regression and combustion of the solid fuel in a hybrid rocket engine is still a subject of extensive research. Its main goal is to determine the physical phenomena that govern the regression process and to create a mathematical model that would accurately determine the rate of solid fuel regression. The most widely used, and at the same time the simplest, is the relation attributed to Marxman and Gilbert [16]:

\[
\dot{r} = a G_{ox}^n
\]

(12)

where \( a \) and \( n \) constants are derived experimentally.

The accuracy of solid fuel regression rate data obtained using this model depends strongly on the accuracy of \( a \) and \( n \) constants, which can be determined in principle only for the engine in which the experiment was carried out. Marxman's model is based on the assumption that for certain intermediate ranges of oxidizer mass flux \( G_{ox} \), the main mechanism causing solid fuel regression is its pyrolysis due to convective heat exchange between the flame and the grains surface [3,16], (Figure 5). With a decrease in the \( G_{ox} \) mass flux, the convective heat transfer coefficient also decreases, which results in an increased role of radiation - this mechanism also applies to fuels producing solid particles as a result of incomplete combustion, e.g. metallic additives, as
Numerical modelling of the hybrid rocket engine performance

noted by Marxman in a later paper [17]. Under such conditions, the regression rate becomes closely related to the pressure $P_{ch}$ in the chamber. Conversely, at high mass flux values, the engine approaches the flooding limit at which the flame may be extinguished under oxidant rich conditions and for insufficient time to completely mix and burn the propellant.

The process of fuel combustion under these conditions has a much more complex nature dependent, among others, on the parameters determining the full kinetics of the chemical reaction and the flow conditions in the boundary layer [3,6,7]. Regardless, Marxman’s model may be a starting point for the development of more accurate models [6]:

$$
\dot{r} = \frac{0.047}{P^2 \rho_f} \left( \frac{C_f \Delta T_{surf}}{h_f} \right)^{0.25} G_{tot}^{0.8} \left( \frac{h_{opt}}{L} \right)^{0.2}
$$

(13)

Although Eq. (13) has the character of a general relationship illustrating the main phenomena occurring in a hybrid rocket engine, its application to the simulation of a particular rocket engine may generate significant errors. This is due to several facts underlying its creation such as:

- lack of reference to experimental data [7],
- the value of coefficient $n = 0.8$ was determined for heat exchange process over a flat plate [6],
- taking into account only the flow of oxidizer, which underestimates increased the port regression along the grain [5],
- is a 0D model, which prevents accurate simulation of grains other than those with single cylindrical ports (e.g. dynamic connection of multiple ports cannot be predicted) [4–6].

Nevertheless, Eq. (13) can be the basis for creating a more accurate empirical model of solid fuel regression, described by the following expression:

$$
\dot{r}(x) = a_1 \Delta T_{surf}^{0.25} G_{tot}(x)^{n} x^{n_2}
$$

(14)

where:

$$
G_{tot}(x) = \frac{\dot{m}_{inj} + \dot{m}_f(x)}{A_{port}(x)}
$$

(15)

In order to perform numerical calculations using Eq. (14), the combustion chamber (fuel grain), has been divided into a specified number of intervals in which local values of $\dot{m}_f(x)$ (total fuel mass flow rate at a distance $x$ from the beginning of the fuel grain) and $A_{port}(x)$ (cross-sectional area of the port at a distance $x$ from the beginning of the fuel grain) are calculated. Since the values of temperature in the combustion chamber and fuel mass flow rate are interrelated, the exact values are calculated iteratively, until a suitable level of convergence is achieved (Figure 2). Although Eq. (14) requires calibration with experimental data, it allows for more accurate simulation of complex grain geometries combustion than Eq. (13), while providing greater overall accuracy than Eq. (12). While more complex models, better reflecting the mechanism of regression in a hybrid rocket engine exist [3,8–10], their complex structure necessitates a larger amount of experimental data for calibration.

4. Test bench description

In order to validate the mathematical model of the engines operating parameters described in this paper (Figure 2), and to obtain empirical data necessary to assess the model's reliability in the context of its use as a design tool, a prototype hybrid rocket engine with a thrust in the range of 200–300 N and a test bench allowing for measurements of engines key parameters (Figure 6) have been created. To ensure optimal combustion conditions of solid fuels, the engine design includes both pre- and post-combustion chambers, which allow the oxidizer to be uniformly distributed between multiple fuel grain ports (in geometries with multiple ports), and reduce the losses resulting from incomplete fuel combustion. The test bench allowed for the measurements of thrust and mass of the used oxidizer through the use two of load cells connected to digital signal converters. Static pressures upstream of the injector as well as in the oxidizer tank were also measured, using capacitance pressure transducers. The use of a solenoid valve in the oxidizer feed line made it possible to stop the combustion process at a given time, without the need to empty the oxidizer tank completely.
5. Test results, discussion of the results

5.1. Cold flow tests

In order to confirm the adequacy of the mathematical model of oxidizer tank blowdown and the injector orifice flow, a series of cold-flow tests has been carried out during which the total mass of used medium and static pressure in the feed line were measured. All of the tests were performed using an identical oxidizer tank and injector of the following geometry:

- number of orifices: 3
- orifice diameter: 1.4 mm
- orifice length: 5 mm

Due to its similar thermodynamic properties, a series of cold-flow tests using CO₂ has also been performed. The CO₂ and N₂O flow tests were performed on an identical installation. The results of the cold-flow tests of the injection system (Figure 7) show slight deviations between the results obtained from the Dyer flow model of the high vapour pressure oxidizer injector (Figure 7 - continuous lines) and the experimental results (Figure 7 - points). The values of flow coefficients $C_d$ (Table 1) change in a small range around the value of 0.91, whereas for the SPI model they would be strongly dependent on the pressure in the installation (Figure 4). It is worth noting that the real pressure change in N₂O and CO₂ installations is greater than that resulting from the model. This may result from the assumption of adiabaticity of the external tank walls and lack of thermal losses to the environment in the model.

5.2. Fuel combustion tests

Research regarding the regression and combustion of the solid fuel is one of the main issues discussed in this work. As part of the research campaign, multiple engine firings have been carried out using both PVC (polyvinyl chloride) and HDPE (high density polyethylene) as fuel (Figure 8) resulting in extensive data about the engine’s operating parameters being collected. During the tests, the engine thrust and mass of the used oxidizer were measured. Due to the inability to measure regression during the test, measurements were made after its completion and the results were averaged. Examples of engine thrust curves for both PVC and HDPE fuels are shown in Figures 9 and 10, respectively, while the comparison of the results of numerical calculations with the results of selected tests is presented in Table 2.

Figures 9 and 10 show that the calculated characteristics are close to the real ones in terms of both shape and value. It has been found that regardless of the fuel used and the oxidizer/fuel mass ratio, both the thrust and specific impulse values obtained from the model (Table 2) exceed the values obtained directly from the experiment by about 10%. The difference is mainly due to the fact that heat loss is not included in the model and a chemical equilibrium model of combustion is used. The approximate constant value of the error means that it is possible to introduce a combustion efficiency constant that will allow for a more accurate prediction of the actual burn rate. The model does not take into account the impact of a pyrotechnic igniter, the effect of which can be seen in the thrust diagram as a peak in the initial phase of the engine’s operation. Due to the existence of various types of ignition in a hybrid rocket engine, this element has been omitted from the model, which is supposed to serve as a general design tool.

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<th>Test number</th>
<th>Medium</th>
<th>$j_{\text{flow}}$/s</th>
<th>$P_0$/bar</th>
<th>$P_1$/bar</th>
<th>$P_1$/model $m_\text{flow}$/g/s $C_d$ [-]</th>
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<td>42.50</td>
<td>43.10</td>
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<tr>
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<td>N₂O</td>
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<td>33.55</td>
</tr>
<tr>
<td>3</td>
<td>N₂O</td>
<td>5</td>
<td>24</td>
<td>23.50</td>
<td>23.80</td>
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<tr>
<td>4</td>
<td>CO₂</td>
<td>15</td>
<td>48</td>
<td>43.00</td>
<td>45.05</td>
</tr>
<tr>
<td>5</td>
<td>CO₂</td>
<td>15</td>
<td>28</td>
<td>26.50</td>
<td>27.10</td>
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</table>

Figure 7  The mass of used N₂O and CO₂ from time, for different initial tank pressures.

Figure 8  Rocket engine during the test.
For comparison, the thrust curves for the models presented in Eq. (12) and Eq. (13) are also presented. As can be seen, the simple Marxman relation provides an almost identical engine thrust curve to the proposed model. This is to be expected since the ballistic coefficients $a$ and $n$ were obtained with the data from the same test, which is then simulated, and the geometry of the engines allows a relatively stable oxygen/fuel ratio. Operating conditions other than those present in the experimental campaign - e.g. changing combustion temperature from 2327 K to 2885 K (corresponding to an oxygen/fuel shift from 2 to 3.75), may generate regression rate errors of up to 6%. Using the more complex relationship shown in Eq. (13), a lower thrust level than the other models discussed is obtained, and even test results, which is probably due to a gross underestimation of the fuel regression coefficient. The longitudinally averaged regression measured after the test is 6.1 mm with the proposed model predicting 6.4 mm, while Eq. (13) shows only 2.7 mm. Such a discrepancy is most likely caused by the material properties found in the literature (and used in this simulation), differing from the properties of the tested propellant grains.

During the process of designing a hybrid rocket engine, an especially close attention should be paid to the injector design (which affects the mode of oxidizer injection) as it can significantly alter the course of the solid fuel regression process. Figure 11 shows cross-sections of solid fuel (PVC) grains after the tests employing an impinging-jet and parallel orifices injectors. It can be seen that as a result of using the impinging-jet injector, there has been an increase in the regression in the initial section of the port (Figure 11(a)), while in the case of an injector with orifices parallel to the combustion chamber axis an almost uniform combustion has been achieved along the entire length of the port. Consequently, it should be noted that in order to increase accuracy, the model requires validation (correction of $a_2$ constant) for a given type of injector (values presented in Table 2 refer to an injector with parallel orifices).

In a logarithmic plot Figure 12 shows the dependence of regression rate $\dot{r}$ on mass flux $G$ obtained from Eq. (14) for PVC and HDPE based fuels and time-averaged $\dot{r}$ and $G$ values.
obtained from conducted experimental tests. As predicted, the experimental points are arranged on a straight line, indicating that the assumption underlying Eq. (14), i.e. the dominant role of convective heat exchange is correct. It is worth noting that the results from tests of PVC fuel grains reach the lower limit of the model’s range, beyond which radiation can play a greater role in heat exchange. Measured data, despite its repeatability, could not have been used to determine the combustion law coefficients $a_1$, $n$ and $a_2$ - because the obtained points result from the comparison of the initial and final port geometries, their position on the plot is not representative of the actual engine operation. They are however a proof of correctly performed test procedures and roughly show the model’s boundaries. Coefficients $a_1$, $n$ and $a_2$ (Table 3) have been determined iteratively by comparing the amount of fuel used during a given test with the result of numerical simulation, where in each consecutive iteration the coefficients were changed until the smallest possible error has been reached.

### 6. Conclusions

Based on the conducted theoretical and experimental research, a numerical model of a hybrid rocket engine has been created. The developed software combines the speed and simplicity of calculations with accuracy, which makes it an attractive tool for the preliminary design and prediction of the behavior of these engines. Particularly noteworthy is the development of a solid fuel regression model that reflects relatively well the complex process of fuel combustion in hybrid rocket engines. Thanks to this, it was possible to verify and create the fuel grain design, which is a significant problem in these engines.

The results of the simulation of the values of important parameters carried out with the use of the proposed model show good agreement with the experimental values — key parameter such as fuel mass flow rate differ from the experimental results by less than 6% (around 1% for most tests). Predicted thrust levels are also consistent with test data, varying between 7.5% and 12%.

Further research is planned on possible ways to increase the accuracy of numerical calculations, e.g. by using a real gas model to simulate the emptying of an oxidizer tank and by creating a more complex model of solid fuel combustion (e.g. based on detailed chemical reaction mechanisms).

Moreover, further work can be done to enhance the applicability of proposed model and to improve the matching of PVC and HDPE combustion parameters, e.g. performing additional tests for a wider range of mass flux values, to establish the boundaries of convection governed regression. Analysis of model’s scalability can also be studied on larger thrust engines.

### References


